Name______________________________
Date__________ Block________

NOTES - FACTORING

Think of factoring as the opposite of distributing. It's a way to undo distributing in order to change the primary structure from a sum or difference into a product.

I. Factoring Using the GCF

GCF = Greatest Common Factor

→ biggest number that can be multiplied into/is a factor of all of the terms

Example: The GCF of 30 and 20 is 10

30: 6, 5; 6: 2, 3 — 20: 2, 10; 10: 5, 2

1, 2, 3, 5, 6, 10, 15, 30

1, 2, 4, 5, 10, 20

To Factor Using the GCF

1. Find the GCF
2. Divide/pull out the GCF from each piece. Answer will look like GCF ("leftovers")

(3. Check your work by distributing)

Example 1: Factor $4x^2+10x$ GCF = 2X

$2 \cdot 2 \cdot x \cdot x \quad 2 \cdot 5 \cdot x$

$\boxed{2x(2x+5)}$

Example 2: Factor $10x^3y-5x^2y^2+25xy^3$ GCF = 5Xy

$\cancel{5}\,2\,\cancel{x}xx\cancel{y} \quad \cancel{5}\cancel{x}x\cancel{y}y \quad \cancel{5}5\cancel{x}\cancel{y}yy$

$5xy(2x^2-xy+5y^2)$

Example 3: Using factoring to simplify $\dfrac{12ab^2-40a^2b}{4ab}$

$12ab^2$: 4, 3, a, b, b — $40a^2b$: 4, 10, a, a, b; 10: 2, 5

$$\frac{\cancel{4ab}(3b-10a)}{\cancel{4ab}} = \boxed{3b-10a}$$

Example 4: Simplify $\dfrac{2x^2-10x}{12x^2-60x} = \dfrac{2x\cancel{(x-5)}}{12x\cancel{(x-5)}} = \boxed{\dfrac{1}{6}}$

$2x^2$ — $10x$: 5, 2, x

Name________________ Date__________ Block______

II. Factoring Quadratic Trinomials with a=1

$ax^2 + bx + c$

a=1 $x^2 + bx + c$

Shortcut: Find 2 numbers whose product is $a \cdot c$ and whose sum is b.

$ax^2 + bx + c$

Example 1: Factor $x^2 + 11x + 30$

$(x + 6)(x + 5)$

Check: $x^2 + 5x + 6x + 30$

$x^2 + 11x + 30$

a·c 1·30 * 30	+ 11
~~3, 10~~	~~+3~~ X
6, 5	11 ✓
2, 15	

Example 2: Factor $x^2 - 9x - 10$

* -10	+ -9
-10, 1	-9

$(x - 10)(x + 1)$

$= (x + 1)(x - 10)$

Note: These are not the same as $(x+10)(x-1)$ The signs matter!

Example 3: Factor $2x^2 - 8x - 24$

There is a GCF hidden! $2(x^2 - 4x - 12)$

$2(x - 6)(x + 2)$

* -12	+ -4
-6, 2	-4

$x^2 - 5^2$

Example 4: Factor $x^2 - 25$ (This is really $x^2 + 0x - 25$).

This is a special case called a **Difference of Squares**: $a^2 - b^2 = (a-b)(a+b)$

$(x - 5)(x + 5)$

* -25	+ 0
5, -5	

Example 5: Simplify $\frac{x^2 + 5x + 6}{x^2 + 2x - 3}$ $\frac{\cancel{(x+3)}(x+2)}{(x-1)\cancel{(x+3)}} = \frac{x+2}{x-1}$

Name________________________
Date__________ Block________

III. Factoring by Grouping

→ Often these problems have 4 terms

1. Cut the problem in half (or into groups of 2)
2. Factor each half using the GCF. (After you do this, the "leftovers" or other factor from each side MUST be equal!!)
3. Pull the GCFs of each side together to make one factor and bring down the matching "leftovers" as the other factor.

Example 1: Factor $a^3 - 4a^2 + 3a - 12$

$a^2(a-4) + 3(a-4)$

$\boxed{(a^2+3)(a-4)}$

Example 2: Factor $2yx + 6y - x - 3$

$2y(x+3) - 1(x+3)$

$(2y-1)(x+3)$

Example 3: Factor $2x^2 - x + 6x - 3$

$x(2x-1) + 3(2x-1)$

$(x+3)(2x-1)$

Name________________ Date__________ Block_______

IV. Factoring Quadratic Trinomials with $a \neq 1$

$ax^2 + bx + c$

1. Find 2 numbers whose product is $a \cdot c$ and whose sum is b.
2. Use those 2 numbers from step 1 to split apart the trinomial into 4 pieces
3. Factor by grouping

$ax^2 + bx + c$

Example 1: Factor $2x^2 + 17x + 21$

$2x^2 + 3x + 14x + 21$

$x(2x+3) + 7(2x+3)$

$\boxed{(x+7)(2x+3)}$

$\times$ 2(21) = 42	+ 17
6, 7	13 ✗
2, 21	23 ✗
3, 14	17 ✓

Example 2: Factor $3x^2 + x - 2$

$3x^2 + 3x - 2x - 2$

$3x(x+1) - 2(x+1)$

$\boxed{(3x-2)(x+1)}$

3(-2) = -6	1
3, -2	1 ✓

V. Factoring Special Cases

1. **Difference of Squares:** $a^2 - b^2 = \underline{(a-b)(a+b)}$

   Example: Factor $4x^2 - 9y^2$

   $(2x)^2 - (3y)^2 = (2x-3y)(2x+3y)$

   a b

2. **Difference of Cubes:** $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

   Example: Factor $x^3 - 8$ (2^3)

   $a = x \quad b = 2 \quad (x-2)(x^2 + 2x + 4)$

3. **Sum of Cubes:** $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

   Example: Factor $8x^3 + 125$

   $a = 2x \quad b = 5 \quad (2x+5)(4x^2 - 10x + 25)$